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(U) Radiation Hydrodynamics in the Ristra Project's Symphony code: Initial release

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Abstract

One of the recent focuses in the next-generation code (NGC/Ristra) project at LANL is to develop a cell-centered Lagrangian radiation-hydrodynamics code under FleCSI framework [1]. The FleCSI framework provides a unified data and execution model in order to insulate production code development from the uncertainty of evolving runtime and architectures. We have developed the code, Symphony, which incorporates the hydrodynamics code, FleCSALE [3], and the radiation transport code, Puno.

This memo is intended to describe the coupling of radiation and hydrodynamics within the Ristra Project's code Symphony.

1 Introduction

Accurate modeling of radiation-hydrodynamics problems is important for many high-energy density physics (HEDP) applications such as Inertial Confinement Fusion (ICF) and astrophysical problems. In such applications, the system generally consists of both optically thick and thin media. Thus, because of the presence of thin media, the kinetic description of the radiation transport equation is desired.

There are several (non-UCNI) radiation-hydrodynamics codes with a high-order radiation transport (Table 1). In general, astrophysical application codes tend to use Eulerian hydro scheme, while ICF application codes often use Lagrange/ALE hydro schemes (xRage [10] is an exception, which is currently used for many ICF applications).

Code	HYDRO	RAD	Institution	Ref.
TROLL	ALE	IMC	CEA (France)	[15]
TORUS	Eulerian	IMC	UK	[11]
FLASH	Eulerian	IMC	U of Chicago	[2]
BHLIGHT	Eulerian MHD	MC	U of Illinois	[25]
MULTI	Lagrange	SN	Germany	[24, 23, 22]
SARA	Lagrange	SN	Spain	[12]
ZEUS-2D	Eulerian	SN	U of Illinois	[26]
ATHENA	Eulerian	SN	U of Illinois	[4, 13]
VULCAN	Eulerian(Lag+remap)	SN	Princeton U	[16]

One of the recent focuses in the next-generation code (NGC/Ristra) project at LANL is to develop a cell-centered Lagrangian and ALE (Lagrange+Remap) radiation-hydrodynamics code under the FleCSI framework [1]. We discuss a detailed Lagrangian radiation-hydrodynamics algorithm implemented in the Symphony code.

2 Lagrangian Radiation Hydrodynamics Equations

We are interested in solving the following Lagrangian hydrodynamic equations:

$$\rho\frac{D\mathbf{u}}{Dt} + \nabla p = \frac{1}{c}\left(\sigma_t\mathbf{F} - \mathbf{D}_F\right), \tag{1}$$

$$\rho\frac{D\mathcal{E}}{Dt} + \nabla\cdot(\mathbf{u}p) = -\sigma_p acT^4 + \sigma_E cE - D_E, \tag{2}$$

$$\rho\frac{De}{Dt} + p\nabla\cdot\mathbf{u} = -\sigma_p acT^4 + \sigma_E cE. \tag{3}$$

Here $\mathcal{E} = e + u^2/2$ is the total specific material energy, $\rho, e, \mathbf{u}, p$ are the density, specific internal energy, velocity and pressure, respectively. The term, $\sigma_E cE - \sigma_p acT^4$, is the energy exchange term between radiation and material due to absorption-reemission physics, and D_E and $\mathbf{D}_F$ are the material

motion correction terms [18],

$$D_E = -\sigma_t \boldsymbol{\beta} \cdot \left[\mathbf{F} - \left(\frac{4}{3} - \theta \right) E\mathbf{u} \right], \tag{4}$$

$$\mathbf{D}_F = \left(\frac{4}{3} - \theta \right) \sigma_t E\mathbf{u}. \tag{5}$$

where $\boldsymbol{\beta} = \mathbf{u}/c$ is the ratio between the material velocity and the speed of light, and $E = (1/c)\int_{4\pi}\int I d\nu d\Omega$, $\mathbf{F} = \int_{4\pi}\int \boldsymbol{\Omega} I d\nu d\Omega$ are the (gray) radiation energy density and radiative flux, respectively. θ is a free parameter whose value depends on the hydrodynamics scheme employed. For example, $\theta = 0$ is used in original Morel's simplified material motion correction [19] and $\theta = 4/3$ is proposed for Eulerian radiation hydrodynamics with Implicit Monte Carlo (IMC) [18]. This form of the material motion correction possesses desired characteristics in a non-relativistic limit, such as (a) total energy and momentum conservation, (b) preservation of an equilibrium solution, and (c) an equilibrium diffusion limit to $O(u/c)$ [19].

A generalized, simple material-motion corrected TRT equation proposed by Lowrie and Wollaber [18] is

$$\frac{1}{c}\frac{\partial I_g}{\partial t} + \boldsymbol{\Omega} \cdot \nabla I_g + \nabla \cdot (\theta \boldsymbol{\beta} I_g) + \sigma_{t,g} I_g = \sigma_{a,g} B_g + \frac{\sigma_{s,g} c E_g}{4\pi} + \frac{1}{4\pi}\left[D_{E,g} + 3\boldsymbol{\Omega} \cdot \mathbf{D}_{F,g} \right], \tag{6}$$

where $I_g(\mathbf{r}, \boldsymbol{\Omega}, t)$ is the specific angular intensity for frequency group g, and $\sigma_{t,g}$, $\sigma_{a,g}$, and $\sigma_{s,g}$ are the total, absorption and scattering opacities, respectively. For Lagrangian radiation-hydrodynamics, $\theta = 1$ is a natural choice because it takes care of the advection term [18]. Using the following relationships:

1. conservation of mass:

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{u} = 0, \tag{7}$$

2. definition of total derivative:

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + \mathbf{u} \cdot \nabla f, \tag{8}$$

yields TRT eqn in the Lagrangian frame:

$$\frac{1}{c}\rho \frac{D}{Dt}\left(\frac{I_g}{\rho} \right) + \boldsymbol{\Omega} \cdot \nabla I_g + \sigma_{t,g} I_g = \sigma_{a,g} B_g + \frac{\sigma_{s,g} c E_g}{4\pi} + \frac{1}{4\pi}\left[C_{E,g} + 3\boldsymbol{\Omega} \cdot \mathbf{C}_{F,g} \right]. \tag{9}$$

We have used the following notation:

$$C_{E,g} = D_{E,g}(\theta = 1) = -\sigma_{t,g}\boldsymbol{\beta} \cdot \left[\mathbf{F}_g - \frac{1}{3}E_g\mathbf{u}\right], \tag{10}$$

$$\mathbf{C}_{F,g} = \mathbf{D}_{F,g}(\theta = 1) = \frac{1}{3}\sigma_t E_g \mathbf{u}, \tag{11}$$

for the material motion correction terms. Finally, we arrive at the following (gray) moment equations by taking the first two angular moments of Eq. 9 and summing over all the frequency groups,

$$\rho\frac{D}{Dt}\left(\frac{E}{\rho}\right) + \nabla \cdot \mathbf{F} + \sigma_E cE = \sigma_p acT^4 + C_E, \tag{12}$$

$$\frac{1}{c}\rho\frac{D}{Dt}\left(\frac{\mathbf{F}}{\rho}\right) + c\nabla \cdot \mathbb{E}E + \sigma_R\mathbf{F} = \mathbf{C}_F \tag{13}$$

where $\mathbb{E}$ is the Eddington tensor. Our radiation-hydrodynamics system consists of Eqs. 1, 2, 3 and 9.

3 Design Concept of Symphony

Symphony code, developed under NGC/Ristra project at LANL, has a unique design concept which distinguishes from previously developed radiation hydrodynamics codes, such as xRage [10]. Symphony is viewed as a collection of single physics codes, rather than a hydrodynamics code with radiation transport capabilities.

Fig. 1 shows the organization of the Symphony code. As can be seen, Symphony consists of four main submodules. FleCSI [1] is the overall computational framework that handles parallel task-executions. FleCSI-SP is the mesh specialization, providing an interface to connectivities for each of the mesh entities (e.g., cells/faces/vertices/corners). FleCSALE [3] is a gas dynamics code, with both Lagrangian and Eulerian hydrodynamics options. Finally, Puno is the radiation transport solver. While, the core part of Puno is a gray radiation transport solver with a P1 approximation, Puno has also the capability of obtaining high-fidelity, multigroup radiation transport solutions. Currently, an interface to Capsaicin S_N radiation transport library is available to obtain transport solutions. Puno has the option of using either the HOLO algorithm [21] or a stand-alone linearized transport through its coupling to Capsaicin.

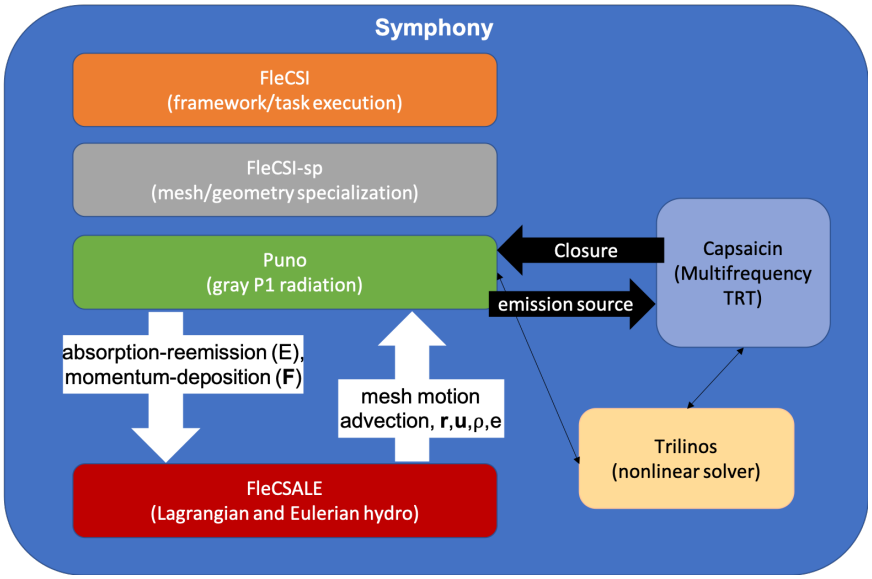

Figure 1: **Symphony code organization**

FleCSI-based physics application codes consist of a collection of "tasks", which enables more concurrent, task-based parallelism. With this task-based code development paradigm, the key roles of Symphony are:

1. to decide how to execute a time step, (e.g., ordering of tasks)
2. to implement tasks describing coupling terms (e.g., material-motion correction, energy-deposition)
3. to set-up the problem, and
4. to post-process the solutions.

Since each physics operator is encapsulated as a small task, in theory, it is relatively straightforward to test different time-integration and coupling schemes between radiation and hydrodynamics. The items 1 and 2 are specifically important features of Symphony, enabling task-based parallelism as well as minimizing data exposure to a single-physics third-party library (TPL), such as the Capsaicin transport code. The next section describes details of the current implementation.

4 Description of Methods/Algorithm required for coupling

As stated earlier, Symphony consists of two single-physics packages, Puno (for radiation transport) and FleCSALE (for hydrodynamics). We first describe how these single-physics packages solve the equations.

4.1 Radiation Transport Solver Strategy

Puno is capable of solving the following multifrequency TRT equations:

$$\frac{1}{c}\frac{\partial I_g}{\partial t} + \mathbf{\Omega}\cdot\nabla I_g + \sigma_{t,g} I_g = \sigma_{a,g} B_g + \frac{\sigma_{s,g} c E_g}{4\pi}. \tag{14}$$

$$\rho\frac{de}{dt} = -\sigma_p a c T^4 + \sigma_E c E. \tag{15}$$

Note that the radiation transport takes into account stiff absorption-reemission physics. Due to the stiff nonlinearity of the absorption-reemission physics and a large dimensionality of the radiation transport equation, obtaining an accurate solution of the radiation hydrodynamics system is a difficult task.

A popular strategy for solving Eqs. 14 and 15 is via linearization of the reemission source [6], which is equivalent to taking a single Newton-step by transforming reemission to an effective scattering. Because of stiffness of the effective scattering term in the optically thick regions, generally, the solution of the linearized TRT equation needs to be accelerated, via subspace methods such as linear multifrequency gray (LMFG) acceleration [14] or hybrid methods such as discrete diffusion Monte Carlo (DDMC) [5].

In order to remedy computational demands on the radiation transport solver, we have recently developed a High-Order, Low-Order (HOLO) algorithm [21]. The HOLO algorithm is a moment-based acceleration method, which employs a discretely-consistent LO system to accelerate the solution of the stiff kinetic systems. Taking the first two angular moments, and summing over all frequency groups yields the following LO system[21],

$$\frac{\partial E}{\partial t} + \nabla\cdot\mathbf{F} + \sigma_E c E = \sigma_p a c T^4, \tag{16}$$

$$\frac{1}{c}\frac{\partial \mathbf{F}}{\partial t} + \frac{c}{3}\nabla E + \sigma_R \mathbf{F} = \boldsymbol{\gamma} c E, \tag{17}$$

$$\rho\frac{de}{dt} + \sigma_p a c T^4 = \sigma_E c E. \tag{18}$$

Note that the closure Eq. 17 is expressed in terms of P1 + consistency term ($\boldsymbol{\gamma} cE$). The nonlinear absorption-reemission term is directly solved in this low-dimensional system without linearization. The nonlinearly-consistent reemission source is then projected to the HO system. The consistency term $\boldsymbol{\gamma}$ can be set to zero when the HO transport is absent, then the LO system will reduce to the standard, gray P1 radiation equations.

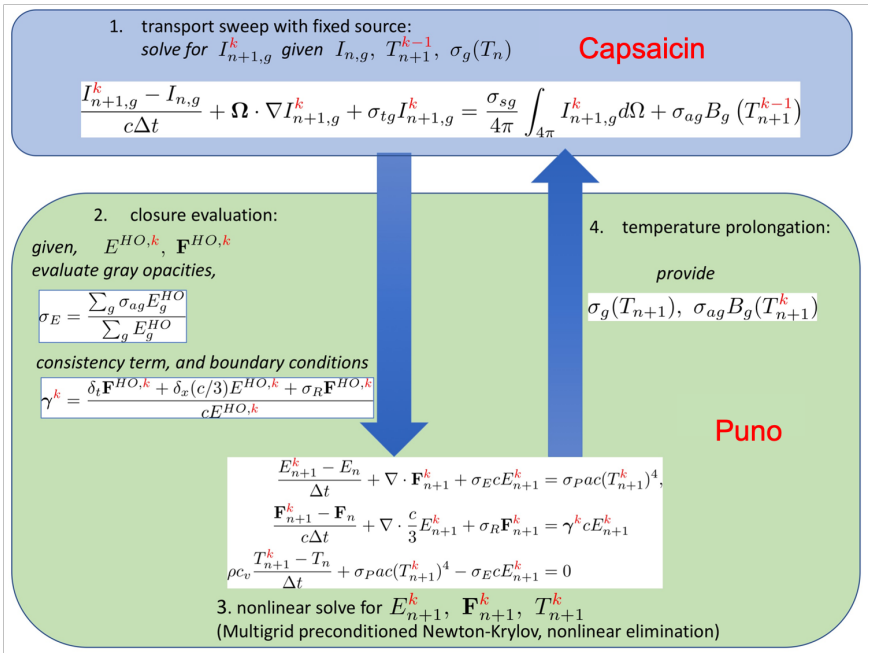

Figure 2: **Puno code organization**

Fig. 2 depicts how Puno integrates the Sn transport code Capsaicin. As can be seen from Fig. 2, Capsaicin's task is to perform transport sweeps with given reemission source (step 1). Using the angular moments of the HO solution, Puno evaluates the weighted gray opacities, the consistency term and the boundary conditions (step 2). After solving the nonlinear LO system (step 3), Puno evaluates a new reemission source, which is used in Capsaicin for the next transport sweep (step 4).

Although Puno was initially developed for use of the HOLO algorithm, it is relatively straightforward to include the stand-alone TRT solver capability because the two Capsaicin interfaces, *Poblano* (HOLO interface) and *Manzano* (linearized TRT interface), are similar. Currently, there are two FleCSI

Algorithm 1: HOLO-SN iterations in Puno

```
Set initial condition I_0, T_0, E_0, F_0 ;
Set n = 0, t_n = 0;
while t_n < t_final do
    t_{n+1} = t_n + Δt_n ;
    set k = 0 ;
    set T^0_{n+1} = T_n ;
    while HOLO iterations not converged do
        k = k + 1;
        HO solve ;
        Using I_n, T^{k-1}_{n+1} perform transport sweeps to obtain
         I_{n+1}, E_HO, F_HO;
        Closure Evaluation;
        Using E_HO, F_HO, evaluate σ_E, σ_P, σ_R, γ and boundary
         condition;
        LO solve ;
        Using σ_E, σ_P, σ_R, γ, solve Eqs. 16-18 for E_{n+1}, T^k_{n+1}
    end
    Update solutions for next time-step;
end
```

tasks (i.e., set-up and solve tasks) when using Capsaicin. The setup task is identical between HOLO and linearized TRT, thus, with a simple swapping of the solve tasks, Puno can also directly solve the linearized form of the TRT equation.

4.2 Hydrodynamics Solver Strategy

FleCSALE in Lagrangian mode solves the following set of equations:

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\mathbf{u} = 0, \tag{19}$$

$$\rho\frac{D\mathbf{u}}{Dt} + \nabla p = 0, \tag{20}$$

$$\rho\frac{D\mathcal{E}}{Dt} + \nabla\cdot(\mathbf{u}p) = 0, \tag{21}$$

The hydro step consists of: (1) computing vertex velocities by solving a

multi-dimensional Riemann problem; (2) evaluating the corner forces; and (3) applying the appropriate corner forces to update the cell-centered hydro variables. FleCSALE uses RK2 (predictor-corrector) as a default time-stepping scheme.

Algorithm 2: Lagrangian Hydrodynamics in FleCSALE

Set initial condition $I_0,\ T_0,\ E_0, \mathbf{F}_0$;
Set $n = 0$, $t_n = 0$;
while $t_n < t_{final}$ **do**
  Predictor Step ;
  evaluate corner forces Fpc_n from previous state ;
  compute numerical fluxes $dUdt_n$ for each cell from Fpc_n ;
  evaluate time-step Δt_n;
  move mesh to $t_{n+1/2}$, using $\mathbf{u}_n,\ \Delta t_n/2$, to get $\mathbf{r}_{n+1/2}$;
  update solution to $t_{n+1/2}\ \rho_{n+1/2},\ u_{n+1/2},\ \mathcal{E}_{n+1/2}$, using $dUdt_n$;
  update $P_{n+1/2},\ T_{n+1/2}$ from $\rho_{n+1/2},\ \mathcal{E}_{n+1/2}$;
  Corrector Step ;
  evaluate corner forces $Fpc_{n+1/2}$;
  evaluate fluxes $dUdt_{n+1/2}$;
  move mesh to t_{n+1}; $\mathbf{u}_{n+1/2},\ \Delta t_{n+1},\ \mathbf{r}_{n+1/2}$;
  update solution $\rho_{n+1},\ \mathbf{u}_{n+1},\ \mathcal{E}_{n+1}$ from $dUdt_{n+1/2}$;
  update $P_{n+1},\ T_{n+1}$ from $\mathcal{E}_{n+1}, \rho_{n+1}$;
  $t_{n+1} = t_n + \Delta t_n$;
  $n = n + 1$;
  Update solutions for next time-step;
end

4.3 Radiation Hydrodynamics Solver Strategy

Currently, Symphony solves a radiation hydrodynamics problem via the operator-splitting between hydrodynamics and radiation during each time step, as suggested in [27, 28, 22, 2, 9]. Each time-step consists of the explicit hydrodynamics step followed by the material motion correction step, and finishes with the implicit radiation step.

We may express radiation-hydrodynamics equations in the following gen-

eral operator notation:

$$\frac{\partial \mathbf{U}}{\partial t} = \mathcal{F}_{hydro}(\mathbf{U}) + \mathcal{F}_{mmc}(\mathbf{U}) + \mathcal{F}_{rad}(\mathbf{U}) \tag{22}$$

where $\mathbf{U}$ is the state variables, and $\mathcal{F}(\mathbf{U})$ is the corresponding (non-)linear operator. Generally, we solve Eq. 22 via operator-splitting, where a "stiff" operator (i.e., $\mathcal{F}_{rad}$) is solved implicitly, while the rest of the operators (e.g., $\mathcal{F}_{hydro}$, $\mathcal{F}_{mmc}$) are solved explicitly. This implicit-explicit (IMEX) operator-splitting results in the following update for the state variables $\mathbf{U}$.

$$\mathbf{U}^{n+1} = \mathbf{U}^{n} + \Delta t \mathcal{F}_{hydro}(\mathbf{U}^{n}) + \Delta t \mathcal{F}_{mmc}(\mathbf{U}^{n}) + \Delta t \mathcal{F}_{rad}(\mathbf{U}^{n+1}) \tag{23}$$

Eq. 23 can be solved by the following steps:

$$\mathbf{U}^{*} = \mathbf{U}^{n} + \Delta t \mathcal{F}_{hydro}(\mathbf{U}^{n}), \tag{24}$$
$$\mathbf{U}^{**} = \mathbf{U}^{*} + \Delta t \mathcal{F}_{mmc}(\mathbf{U}^{n}), \tag{25}$$
$$\mathbf{U}^{n+1} = \mathbf{U}^{**} + \Delta t \mathcal{F}_{rad}(\mathbf{U}^{n+1}) \tag{26}$$

Using the above operator-splitting, a radiation-hydrodynamics time-step consists of the following:

1. Lagrangian hydro step (predictor-corrector) (**FleCSALE**):

$$\mathbf{r}_{n+1} = \mathbf{r}_{n} + \mathbf{u}_{n+1/2}\Delta t \tag{27}$$
$$\mathbf{u}_{h} = \mathbf{u}_{n} - \frac{\Delta t V}{m}\left(\nabla p_{n+1/2}\right) \tag{28}$$
$$e_{h} = e_{n} - \frac{\Delta t}{m}\left(V \nabla \cdot \mathbf{u}p\right)_{n+1/2} \tag{29}$$

2. Angular intensity adjustment due to mesh motion (**Symphony**):

$$I_{h} = I_{n}\frac{\rho_{n+1}}{\rho_{n}} \tag{30}$$

3. Momentum deposition(**Symphony**):

$$\mathbf{u}_{n+1} = \mathbf{u}_{h} + \frac{\Delta t}{\rho c}\left(\sigma_{t}\mathbf{F}_{n} - \mathbf{C}_{F}^{n}\right) \tag{31}$$

4. Material motion correction(**Symphony**):

$$\frac{I_m - I_h}{c\Delta t} = \frac{C_E^n + 3\mathbf{\Omega} \cdot \mathbf{C}_F^n}{4\pi} \tag{32}$$

5. Remap[1] (**Portage**):
   Remap cell averaged state variables to new mesh.

$$I_{remap} = \frac{V_{old}}{V_{new}} \frac{\int_{V_{new}} I_{remap} dV}{\int_{V_{old}} I_m dV} I_m, \tag{33}$$

6. Radiation step (**Puno**):

$$\frac{I_{n+1} - I_m}{\Delta t} + \mathbf{\Omega} \cdot \nabla I_{n+1} + \sigma I_{n+1} = \sigma B, \tag{34}$$

$$\frac{e_{n+1} - e_h}{\Delta t} + \sigma a c T_{n+1}^4 - \sigma c E_{n+1} = 0 \tag{35}$$

Symphony uses Portage [7] for multimaterial remapping. Currently Symphony is capable of a running radiation hydrodynamics problem using a Lagrange+remap. When the Lagrange+remap algorithm is employed, the remap step is performed before the implicit radiation step, and we remap cell-averaged state variables. The corner-based specific intensities are rescaled by the ratio of cell-averaged specific intensities (Eq. 33,)in order to conserve the radiation energy in the system.

Fig. 3 depicts the organizations of the tasks in Symphony with the HOLO radiation step. Steps 2, 3, 4 and 6 account for radiation-hydrodynamics coupling, and steps 3 and 4 can be performed in any order. Note that all tasks associated with radiation-hydrodynamics coupling are implemented inside Symphony. Taking care of all the radiation-hydrodynamics coupling terms within Symphony enables use of a unified interface for the HOLO algorithm, regardless of the type of hydrodynamics schemes employed (i.e., Lagrangian or Eulerian, or lack thereof). As can be seen from step 1 in Fig. 2 and step 5a in Fig. 3, only difference among these methods is how to adjust (or not adjust) specific intensity, I, and its angular moments, to take into account the mesh motion and the momentum depositions. Furthermore, a similar set of tasks are required even if the high-order radiation transport is absent.

[1] when present

We emphasize that this form of operator-splitting is convenient for developing a modular radiation-hydrodynamics code because the hydrodynamics and radiation steps remain exactly the same as the one for the single-physics system. Performing the radiation step at the end of each time step guarantees the proper material-radiation energy balance and satisfies Eq. 23. This holds true even when the remap is present.

Similar to the stand-alone radiation solve, we have two options for performing the radiation step, either solving the TRT equation via the HOLO algorithms (Fig. 3) or linearized SN (Fig. 4) with a simple swapping of the transport tasks. In both cases, the radiation-hydrodynamics coupling tasks (i.e., mesh-motion, momentum deposition, material motion correction, and energy deposition) are the same.

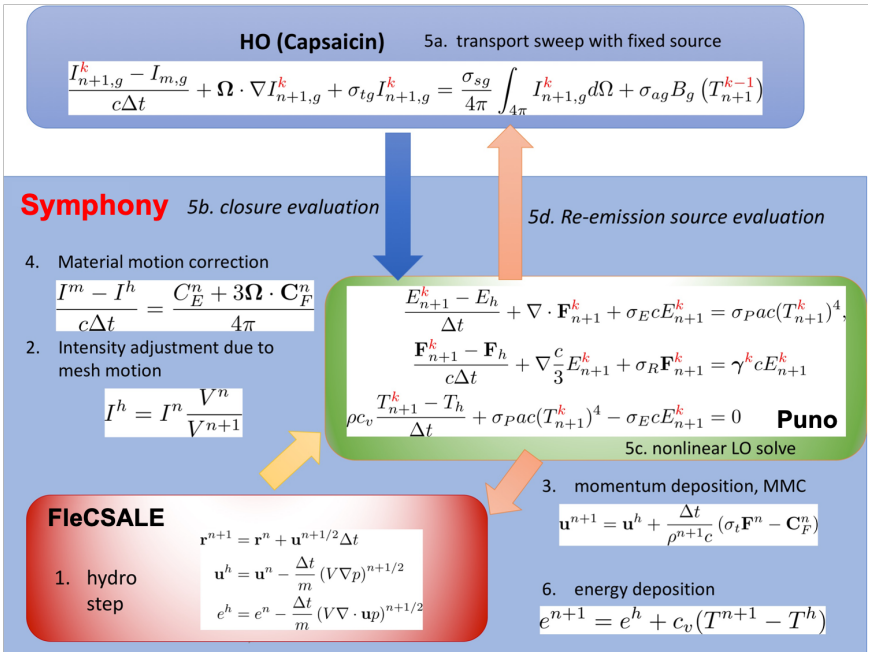

Figure 3: **Radhydro algorithm in Symphony**

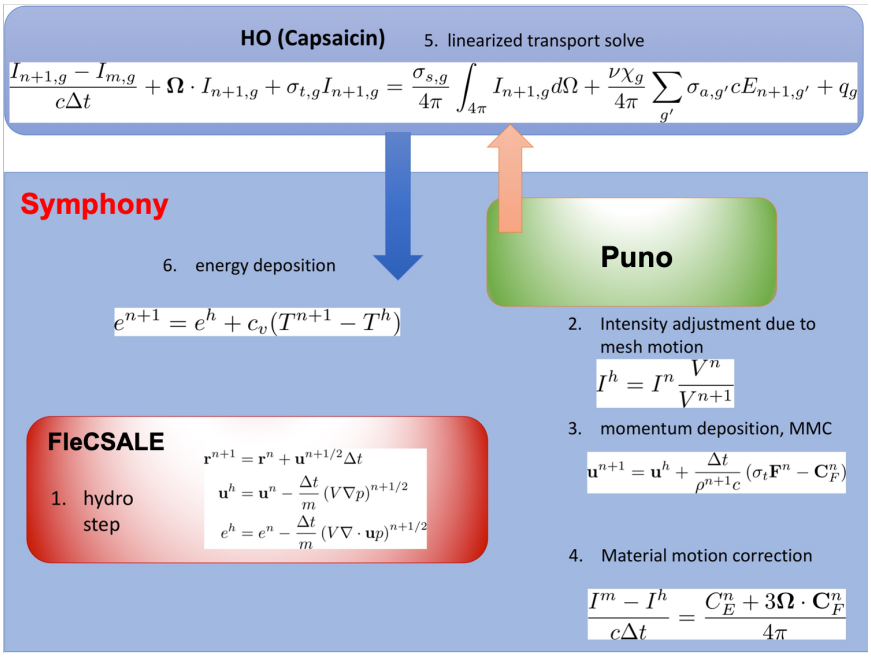

Figure 4: **Radhydro algorithm in Symphony with Linearized TRT**

4.4 Multi-material Treatment

Lastly, we describe a multi-material treatment used in Symphony. Currently, FleCSALE is capable of running multi-material hydrodynamics, while Puno uses a "mixture" or homogenized material in each cell[2]. As described in the previous sections, coupling between the hydrodynamics and the radiation transport occurs through the momentum and energy exchange terms. When mixed material cells exist, one needs to define the following mixed-cell quantities:

1. opacities
2. specific heat,
3. material temperature in mixed cell, and

[2]Each computational cell has one material temperature and radiation energy density

4. distribution of energy deposition to each material due to absorption-emission.

To evaluate a mixed opacity $\sigma\,[cm^{-1}]$, we use a volume fraction, $V_i = \frac{v_i}{V}$, weighted opacity,

$$\sigma = \sum_i V_i \sigma_i, \tag{36}$$

$$= \sum_i \frac{v_i}{V} \sigma_i \tag{37}$$

This formulation is equivalent to a mass fraction, $M_i = \frac{\rho_i v_i}{\rho V}$, weighted specific opacities $\kappa\,[cm^2/g]$.

$$\kappa = \sum_i M_i \kappa_i \tag{38}$$

$$= \sum_i \frac{\rho_i v_i}{\rho V} \frac{\sigma_i}{\rho_i} \tag{39}$$

Hence,

$$\rho\kappa \equiv \sigma = \sum_i \frac{v_i}{V} \sigma_i \tag{40}$$

Note that we evaluate material opacities of each material in cell, $\sigma_i(\rho, T)$, using "bulk (mixed)" density and temperature. "bulk" material temperature and specific heat, $c_v\,[erg/g - eV]$ of a cell are evaluated as

$$T = \sum_i M_i c_{vi} T_i / c_v, \tag{41}$$

$$c_v = \sum_i M_i c_{vi} \tag{42}$$

After a radiation step, an energy deposition due to absorption-emission physics must be redistributed to each material. For a mixed (bulk) specific internal energy, we have the following relations:

$$e^{n+1} - e^h = c_v \left(T^{n+1} - T^h\right) = \Delta e \tag{43}$$

For each material in a cell, we use a mass fraction to redistribute the bulk energy deposition as follows,

$$e_i^{n+1} = e_i^h + c_{vi} \left(T^{n+1} - T^h\right), \tag{44}$$

5 Proposed Numerical Tests to be performed

To verify the correctness of implementation and validity of our radiation-hydrodynamics algorithm, we plan to execute a series of numerical tests. There are three regimes of interest in non-relativistic radiation-hydrodynamics problems [26]:

- streaming limit $\frac{\lambda_p}{l} \geq 1$.
- static diffusion limit. $\frac{u}{c} < \frac{\lambda_p}{l}$. Optically thick, "motionless" media $P = \frac{1}{3}E$, $\frac{F}{cE} = O(\frac{\lambda_p}{l})$
- dynamic diffusion limit, $\frac{u}{c} > \frac{\lambda_p}{l}$. Optically thick, moving medium, $P = \frac{1}{3}E$, $\frac{F}{cE} = O(\frac{u}{c})$

To test validity of our algorithm in these three regimes, we first follow a recently proposed, series of 1D small scale problems [20] with increasing complexities. The problems described in [20] are suitable to test algorithmic behavior of the static diffusion limit. In addition to the proposed small scale problems, we will use the Mach 45 radiative shock problem [17] and the dynamic diffusion test [13] for verification of the material-motion corrections and dynamic diffusion terms. We plan to use cross-beam in vacuum [13] and tophat problem [8] for testing streaming limit. Furthermore, when analytical and/or benchmark solutions are not available, we will compare the solutions obtained by the HOLO radiation and linearized TRT options within Symphony. Since these two options share input, problem setup, and postprocess, it is easier to eliminate inconsistency and enables a focus on the difference in numerical implementations.

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Algorithm 3: Operator-split Lagrangian Radiation Hydrodynamics in Symphony with linearized Sn Transport

Set initial condition $I_0,\ T_0,\ E_0, \mathbf{F}_0$;
Set $n = 0$, $t_n = 0$;
while $t_n < t_{final}$ **do**
  1. LAGRANGIAN HYDRO STEP;
  Predictor Step ;
  evaluate corner forces Fpc_n from previous state ;
  compute numerical fluxes $dUdt_n$ for each cell from Fpc_n ;
  evaluate time-step Δt_n;
  move mesh to $t_{n+1/2}$, using $\mathbf{u}_n,\ \Delta t_n/2$, to get $\mathbf{r}_{n+1/2}$;
  update solution to $t_{n+1/2}$ $\rho_{n+1/2},\ u_{n+1/2},\ \mathcal{E}_{n+1/2}$, using $dUdt_n$;
  update $P_{n+1/2},\ T_{n+1/2}$ from $\rho_{n+1/2},\ \mathcal{E}_{n+1/2}$;
  Corrector Step ;
  evaluate corner forces $Fpc_{n+1/2}$;
  evaluate fluxes $dUdt_{n+1/2}$;
  move mesh to t_{n+1}; $\mathbf{u}_{n+1/2},\ \Delta t_{n+1},\ \mathbf{r}_{n+1/2}$;
  update solution $\rho_{n+1},\ \mathbf{u}_h,\ \mathcal{E}_h$ from $dUdt_{n+1/2}$;
  update $P_h,\ T_h$ from $\mathcal{E}_h, \rho_{n+1}$;
  2. INTENSITY ADJUSTMENT DUE TO VOLUME CHANGE;
  using ρ_n, ρ_{n+1} evaluate I_h;
  3. RADIATION MOMENTUM DEPOSITION;
  using $\mathbf{F}_n,\ \mathbf{C}_F^n$, evaluate $\mathbf{u}_{n+1}$;
  4. MATERIAL MOTION CORRECTION;
  using $\mathbf{u}_n,\ \mathbf{u}_{n+1}$ evaluate I_m;
  4b. REMAP STEP;
  5. RADIATION STEP;
  compute $I_{n+1},\ T_{n+1}$ from $I_m,\ T_h$
  6. ENERGY DEPOSITION;
  $e_{n+1} = e_h + \Delta t_n \rho_{n+1} c_v (T_{n+1} - T_h)$;
  update $P_{n+1},\ T_{n+1}$ from $\mathcal{E}_{n+1}, \rho_{n+1}$;
  $t_{n+1} = t_n + \Delta t_n$;
  Update solutions for next time-step;
end